municated to lower layers of the sea. This cause of surface currents is of importance to the theory of movement of those polar waters which, for several months after the winter ice begins to break up, are free from larger wind waves. Deprived of its chief sails, the Labrador current, always sensitive to wind conditions and at times subject to temporary reversal with contrary winds, yet preserves and perhaps exceeds, during the period of ice drift, the average velocity of current flow for the year.

THE DYNAMIC PRINCIPLE OF THE CIRCULATORY MOVEMENTS IN THE ATMOSPHERE.

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The hydrodynamic equations of motion undoubtedly contain the key to the explanation of all atmospheric motions, but we meet with the great difficulty that we can not write the integrals of these equations for the complex conditions occurring in the earth's atmosphere. In order, therefore, to introduce rational dynamic methods into meteorology we must endeavor to devise a method by which we may apply the dynamic principles contained in these equations without integrating the equations themselves. In order to do this, we can scarcely suggest a better path than that followed by von Helmholtz and Kelvin for ideal fluids, when the former developed the laws of vortex motion and the latter developed the mathematically equivalent laws of circulatory movements.

As is well known, we attain the original Helmholtz-Kelvin theorems when we start with the equations of motion for frictionless fluids and supplement these by a restrictive assumption, viz, either that the fluid is homogeneous and incompressible or that the density of the fluid is a function of the pressure only. It is well known that this latter assumption is as far from the truth as the assumption that the atmospheric air is frictionless. These theorems of Helmholtz and Kelvin also show that circulatory and vortex motions can have neither beginning nor ending, and they therefore leave the fundamental question as to the initial formation of these motions undisturbed, so that they have only a very limited application in meteorology. But in order to attain more general theorems that do contain the laws of the formation and annihilation of both circulatory and vortex motions in the atmosphere, we need only follow the same course of reasoning that led to these theorems, starting, however, with assumptions of properties that better represent those of the natural fluids.

These generalizations are best executed step by step, and in doing this we can proceed according to either of the following schemes:

1. With von Helmholtz and Lord Kelvin we start with the equations of motion for frictionless fluids, but in the course of the study we avoid introducing any special limiting assumptions in reference to the density of the fluid.

2. We develop the corresponding theorems by starting from the equations of motion of viscous fluids (namely, those that have internal friction or viscosity).

A rearrangement of the theorems thus obtained will be found to be important in order to bring them into the form most appropriate for the proposed applications.

3. We refer all the theorems to a system of rotating coordinate axes in order that only the circulatory or vortex movements relative to the rotating earth may need to be considered in the proposed applications.

The first of these three general methods is beyond comparison the most important of all. Through it we attain to an exhaustive treatment of the primary causes of motion in the atmosphere, which, as is well known, are to be sought in the differences of density that depend upon the temperature. The second and third general methods only show how the motion already produced is modified in its subsequent course, partly by the "deflecting force of the earth's rotation," which seeks to change the direction of the motion, insofar as we consider the latter relative to the rotating earth, and, in part, by the friction which seeks to smooth out all differences of velocity.

In the following I shall consider exclusively this first general method and its applications to meteorology. In doing so, I shall deduce the theorem as one relating simply to circulation as an extension of Kelvin's mode of presentation. This method has important practical advantages over the mathematically equivalent form, where we start with Helmholtz's conception of a vortex.

At present I shall give the deduction of the theorem in the most elementary form possible, starting out with general dynamic principles and not with the hydrodynamic equations of motion. As to other methods of deduction and other forms of the theorem and other applications than those that are purely meteorological, I will merely refer to my previous publications.[1] I also refer to the memoir by L. Silberstein who first investigated that generalization of Helmholtz's vortex theorem which is now under consideration.[2]

Of the five sections into which this present memoir is divided, the first contains the definition of the term "circulation" as here used and the deduction of the mathematical properties of this conception, so far as they are needed in the subsequent sections. The second section describes a geometrical method of representing the dynamic condition of a fluid that is of equal importance to both the deduction and the application of the theorem. Finally, the third section gives the demonstration of the fundamental dynamic theorem relative to circulation and the two last sections treat of the applications of this theorem to the movements of the atmosphere. I would especially state that in the preparation of these last sections, the explanation and advice of Dr. N. Ekholm have been very useful to me.

I.—CIRCULATION.

Let us consider a continuous chain of fluid particles forming a closed curve. Each of these particles has a definite velocity, U, and the component of this velocity, tangential to the curve, is U_t. By the summation of these latter components, along the curve, we obtain

$$C = \int U_t \, ds, \tag{1}$$

where ds is a line element of the curve. The quantity, C, as found in this manner, we will call the circulation of the curve, s, as was done by Lord Kelvin.[3]

In reference to this conception of the circulation of a fluid curve, it should first be remarked that we may find its value for any given curve in the atmosphere by the observation of the wind. As an example, we may consider a curve which runs along the earth's surface as an arc of the meridian from the pole to the equator and then returns along a similar meridional arc at the altitude of the highest cirrus clouds from the equator to the pole. As elements of the curve we can make use of any appropriate degree of the meridian, and

[1] V. Bjerknes, Ueber die Bildung von Cirkulationsbewegungen und Wirbeln in reibungslosen Flüssigkeiten. Videnskabsselskabets Skrifter, Christiania, 1898. Ueber einen hydrodynamischen Fundamentalsatz und seine Anwendung besonders auf die Mechanik der Atmosphäre und des Weltmeeres. Kongl. Svenska Vetenskapsakademiens Handlingar, Band 31, Stockholm, 1898.

[2] L. Silberstein, Bulletin International de l'Academie des Sciences de Cracovie, 1886.

[3] Sir W. Thomson, On vortex motion. Transactions of the Royal Society of Edinburgh, 1869, § 60. Vol. XXV, p. 248.

as the positive direction of motion along the curve we can choose that which, at the earth's surface, passes from the pole to the equator and in its upper portion passes from the equator to the pole. The east-west component of the wind being perpendicular to this curve does not come into consideration, but only the north-south component directed along the curve. For each degree of the meridian we form the product of the mean average north-south wind component multiplied by the length of the degree and take the sum of the products. In this summation the vertical velocities do not come into consideration, first, because the vertical portion of the curve is inappreciably short in comparison with the horizontal, and, second, because also the vertical velocities are very small in comparison with the horizontal. The proper velocities at the earth's surface are found from the ordinary measurements of the wind; those in the upper regions from observations of the movements of the cirrus clouds. Dividing by the total length of the curve we obtain the mean velocity in the direction tangential to the curve.

The value, C, of the circulation for this curve can be considered as a measure of the circulatory movement of the atmosphere between the pole and the equator. The momentary value of the circulation, as found by using simultaneous observations, as well as the mean value for longer periods of time, such as months, seasons, or whole years can be found by this method.

A simple property of integrals of the form (1) will come into play both in the deduction of the fundamental dynamic theorem as also in all practical applications. Consider a series of curves, 1, 2, 3, . . . n adjacent to each other, as in fig. 1, and let C_1, C_2, C_3, . . . C_n be the corresponding values of the linear integrals, equation 1. Take the sum of all these linear integrals, assuming the same positive direction of circulation for each of the curves, then, as we can see by studying fig. 1, the linear integrals along every part of the curve that has two curves in common eliminate and disappear; for in the summation the corresponding linear integrals enter once positively and once negatively. The result of the summation is, therefore, simply equal to the linear integral, C, along the outer boundary, viz.,

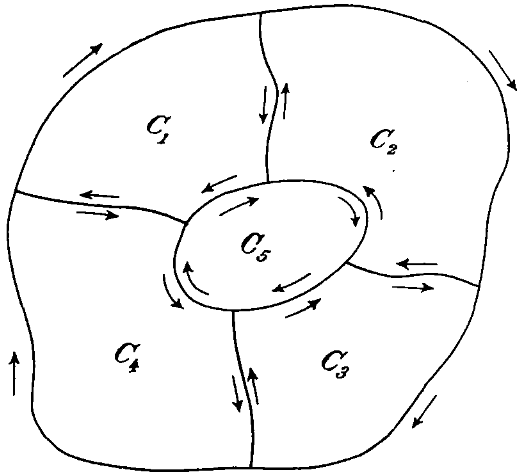

FIG. 1.

(2) $$C = C_1 + C_2 + C_3 + \ . \ . \ . \ + C_n,$$

or, in other words:

The sum of the linear integrals along a series of adjacent curves is equal to the linear integral along the common exterior boundary.

The circulation between the pole and the equator, mentioned above as an illustration, can, therefore, be considered as the sum of a series of smaller individual circulations, of which, one, for example, may be the circulation in the trade wind region proper; another, the circulation in the middle latitudes; and a third, the circulation in the polar region. These individual circulations can be studied quite independently, and afterwards we can obtain the total circulation between the pole and the equator by simple summation.

It will now be our problem to find the law according to which the circulation of any given chain of particles of air changes with the time, under any given dynamic conditions. In order to prepare for the solution of this problem it will be appropriate to investigate the mathematical expression for the change of circulation with time.

It will be most convenient to use the rectangular coordinates x, y, z. Let dx, dy, dz be the projections upon these axes of the linear element of the curve ds, and let U_x, U_y, U_z be the projections upon the same axes of the velocity U of the point on the curve represented by x, y, z, then the expression (1) for the linear integral becomes

$$C = \int \left(U_x\, dx + U_y\, dy + U_z\, dz. \right)$$

If we differentiate this expression with reference to the time, then we must remember that the curve is in motion, so that not only the velocity components U_x, U_y, U_z, but also the projections dx, dy, dz of the linear element ds, vary with the time. Therefore such differentiation gives

$$\frac{dC}{dt} = \int \left(\frac{dU_x}{dt} dx + \frac{dU_y}{dt} dy + \frac{dU_z}{dt} d \ \right)$$
$$+ \int \left(U_x \frac{d}{dt} dx + U_y \frac{d}{dt} dy + U_z \frac{d}{dt} dz. \right)$$

We will first seek the value of the second line on the right-hand side of this equation. The differentiation with reference to time, indicated by $\frac{d}{dt}$ and the operation by which we have separated the curve into linear elements, in order to accomplish an integration along the curve up to a definite point of time, are entirely independent operations. We can, therefore, interchange the order in which these operations are performed and can write this second line as follows:

$$\int \left(U_x\, d\, \frac{dx}{dt} + U_y\, d\, \frac{dy}{dt} + U_z\, d\, \frac{dz}{dt} \right).$$

But we know that the differentials of x, y, z, with reference to t, are simply the velocity components U_x, U_y, U_z for the point x, y, z in the curve, viz:

$$\frac{dx}{dt} = U_x\,, \quad \frac{dy}{dt} = U_y\,, \quad \frac{dz}{dt} = U_z\,,$$

so that the above expression becomes

$$\int \left(U_x\, dU_x + U_y\, dU_y + U_z\, dU_z \right)$$

or,

$$\int \frac{1}{2}\, d \left(U^2{}_x + U^2{}_y + U^2{}_z \right).$$

But this is the integral of a total differential, and, therefore, is 0 when it is taken along any closed curve.

In the above expression for the differential of the circulation, C, with reference to the time, there now remains only the first line on the right-hand side, and this has a simple meaning. The differentials of the component velocities, U_x, U_y, U_z are the components of the acceleration V of the point x, y, z of the curve, viz:

$$\frac{dU_x}{dt} = V_x, \quad \frac{dU_y}{dt} = V_y, \quad \frac{dU_z}{dt} = V_z$$

The differential of the circulation with respect to the time is therefore

$$\frac{dC}{dt} = \int \left(V_x\, dx + V_y\, dy + V_z\, dz. \right)$$

That is to say, if we designate by V the component of the acceleration in the direction tangential to the curve, we have

$$(3) \qquad \frac{dC}{dt} = \int V_t \, ds$$

or: *The increase of the circulation of a closed curve in a unit of time is equal to the integral, taken along the curve, of that component of the acceleration that is tangential to the curve.*

In order to find the dynamic law of the change of the circulation with the time, we therefore need only to integrate the component accelerations due to the individual active forces in the direction tangential to the curve. Therefore, all accelerating forces that have a linear integral equal to zero along closed curves are unimportant. This leads us to a very important simplification of our problem, for it is well known that all accelerating forces of a conservative nature have this property. Therefore, in considering the circulation along closed curves in the atmosphere we need never take into consideration the force of gravity, since it is a conservative force.

If at the same time we also, in accordance with our assumptions, omit the consideration of friction and the deflecting force of the earth's rotation, then we shall only have to consider the accelerating force resulting from the pressure of the fluid. The linear integral of this force will be easily determined after we have considered a geometrical presentation of the dynamic conditions in the interior of gaseous or fluid media.

II.—GEOMETRIC PRESENTATION OF THE DYNAMIC CONDITIONS IN LIQUID OR GASEOUS MEDIA.

The distribution of the pressure p in any gas or liquid can be shown with the help of surfaces of equal pressure or isobaric surfaces for which p is constant The gradient G is perpendicular to the isobaric surfaces, and is directed toward the diminishing pressure. If n is the normal to an isobaric surface, and is directed against the increasing value of the pressure (i. e. toward the lower pressure), then the expression for the gradient may be written

$$(4) \qquad G = -\frac{dp}{dn}.$$

It will be especially convenient to draw the isobaric surfaces for pressure differences of one unit. By a convenient choice of units we can always bring it about that the isobaric surfaces shall run close enough to each other to represent the distribution of pressure in the fluid with sufficient completeness.

The acceleration that the gradient communicates to a particle of fluid depends on the inertia, that is to say, on the density of the particle; it is equal to the gradient divided by the density, or, still simpler, it is equal to the gradient multiplied by the specific volume, k, of the fluid particle. In order to be able to express the distribution of the acceleration so far as it depends upon the pressure, it is therefore sufficient to know the distribution of pressure and at the same time that of the specific volume throughout the fluid. This distribution can be expressed with the help of surfaces of equal specific volume, or isosteric surfaces, for each of which k is constant. These surfaces we always think of as drawn for each unit of difference of the specific volumes and, in doing so choose a unit of convenient magnitude such that the surfaces lie sufficiently near to each other, in order to represent the distribution of the specific volume in all portions of the fluid, with satisfactory accuracy.

Following the analogy of the gradient, we can define a vector, B, by the equation

$$(5) \qquad B = \frac{dk}{dn}$$

where n is the normal to an isosteric surface taken positively in the direction of increasing specific volumes. Therefore B is a vector quantity that points in the direction of increasing specific volumes and since the mobility of the fluid increases with the specific volume, we can call B the vector of motion. It will be remarked that in equation (5) we have used the positive sign, whereas in equation (4), defining the gradient, the negative sign occurs. A vector quantity $(-B)$, defined in complete analogy with equation (4), would in general have a direction almost exactly opposite to the direction of the gradient, since with diminishing pressure an increasing specific volume usually follows. On the other hand, the vector of motion, B, has approximately the same direction as that of the gradient, G, and is therefore to be preferred to $-B$ in the applications.

Some general remarks as to the course of the isobaric and the isosteric surfaces are important.

1. It is to be considered that an isobaric surface can never come to an end in the interior of a fluid; it must either reenter into itself or else end at the boundary surfaces of the fluid. The isobaric surfaces in the atmosphere, for instance, either surround the whole earth as closed surfaces, agreeing very closely with the level surfaces of gravitation, or else they end against the surface of the earth which cuts them along the isobaric curves that we draw by means of ordinary barometric observations.

The isosteric surfaces have precisely the same property; they can not end in the interior of a fluid any more than can the isobaric, but they must continue on until they run into themselves or until they end against the bounding surfaces of the fluid. In the atmosphere they have, approximately, the same course as the isobars; the upper isosteres surround the whole earth, whereas the lower ones intersect the earth's surface along the isosteric curves.

A second property of the isobaric surfaces is that two neighboring surfaces, representing different values of the pressure, p, can never intersect each other; throughout their whole course they must be separated from each other by an isobaric layer, which, on its part, has the same fundamental property as the surfaces, namely, either returning into itself or terminating against the boundary surfaces of the fluid. Similarly, the successive isosteric surfaces are separated from each other by corresponding isosteric layers.

These two sets of surfaces together divide the whole space into tubular or prismatic portions, which we may designate as *isobaro-isosteric tubes*. From the properties of the isobars and the isosteric layers that belong to these tubes, it follows that the latter also have this peculiarity that each either runs into itself or terminates at the boundary surfaces of the fluid. If the surfaces are drawn for each unit difference of pressure and of specific volume, we may call the corresponding tubes, *unit tubes*. If we assume that we use the units just mentioned of proper dimensions, then we may consider the corresponding unit tubes as infinitesimal *solenoids*. The cross sections of the larger isobaro-isosteric tubes have the form of curved quadrilaterals; the cross sections of the solenoids are rectilinear parallelograms.

Since the solenoids have this property that they either return into themselves or terminate at the boundary surfaces, therefore, every closed curve in the fluid incloses a definite bundle of solenoids; the number, A, of solenoids in this bundle becomes a simple definite number as soon as the units of specific volume and pressure have been chosen.

III.—DEDUCTION OF THE FUNDAMENTAL DYNAMIC THEOREM RELATIVE TO THE CIRCULATION.

In order to investigate the dynamic conditions necessary for the existence of circulatory movements as a consequence of fluid pressure, we will consider a portion of the fluid so small that within it we may consider the specific volume and the pressure as linear variable qualities. In this portion of the fluid the isobaric surfaces extend as a set of parallel equidistant planes, and the isosteric surfaces as another set of parallel equidistant planes. The solenoids are tubes whose cross sections form a system of parallelograms congruent to each other. Throughout this part of the fluid the gradient will have an invariable magnitude and direction, and this will also be the case with the vector of motion.

If all particles of the portions of the fluid under consideration had had equal specific volumes, then the gradient would have communicated an equal acceleration to all points, and the result of the effect of the gradient during an element of time would have remained a simple pure motion of translation superposed upon the previous velocity of this part of the fluid. But on account of the variability of the specific volume from point to point the different points will take up accelerations of different amounts, in such a way that the lighter portions will move more swiftly than the heavier. Thus therefore, the gradient produces not only a translatory but also a rotatory motion, by virtue of which the fluid masses are turned around the intersections of the isobaric and isosteric surfaces as axes, and in the direction from the vector of motion, B, by the shortest way to the gradient G.

By reason of this rotation of the fluid masses, there results a circulation of all closed curves consisting of particles of fluid. We need consider only plane curves within the small portion of the fluid under consideration. The following rule will determine the direction of the acceleration of circulation that one of these curves experiences:

Project the gradient and the vector of motion on the plane of the curve; then the acceleration of circulation is directed by the shortest route from the projection, B, of the vector of motion toward the projection, G, of the gradient. (See fig. 2.)

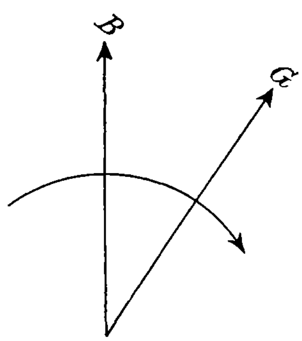

Fig. 2.

In order to find the quantitative law for the resulting acceleration of circulation we recall that according to formula (3) the increase per unit of time in the circulation is proportional to the line integral of the component of the acceleration that is tangential to the curve. We will first seek to determine the value of this line integral of the acceleration for the curve produced by intersection of an isobaro-isosteric tube with any arbitrary plane. This curve has a parallelogrammatic form, fig. 3, two of whose parallel sides, p_0 and p_1, lie in an isobaric plane and two, k_0 and k_1, in an isosteric plane. If h is the distance of the two isobaric planes from each other, then the gradient has the numerical value

$$G = \frac{p_1 - p_0}{h}.$$

Since the gradient is perpendicular to the two isobaric sides of the parallelogram, it can cause no acceleration in a direction tangential to these lines. But the gradient forms an angle, θ, with the isosteric sides of the parallelogram and consequently produces, in a direction parallel to these lines, the component accelerations $k_1\ G \cos\theta$ and $k_0\ G \cos\theta$. If we refer both these to the same direction of circulation around the curve, $p_0\ k_0\ p_1\ k_1$, then they become

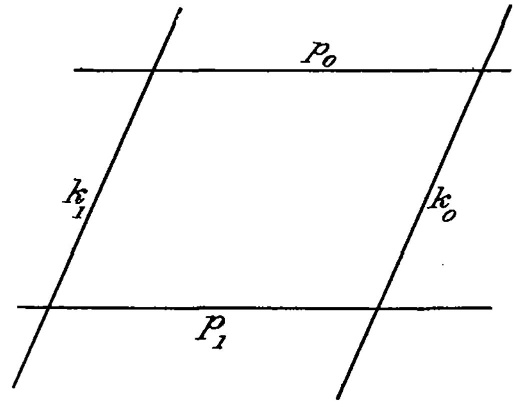

Fig. 3.

$$k_1\ G \cos\theta \text{ und } - k_0\ G \cos\theta.$$

In order to find the value of the line integral we have to multiply these quantities by the length of the corresponding line elements and add the products thus formed. But both sides of the parallelogram have the same length, $\frac{h}{\cos\theta}$, so that we find $(k_1 - k_0)\ Gh$ as the value of the line integral. If we introduce the above value of the gradient, G, this integral becomes

$$(k_1 - k_0)\ (p_1 - p_0).$$

Finally we may specialize by the assumption that the isobaro-isosteric tube under consideration is a solenoid. According to the definition of the solenoid $k_1 - k_0 = 1$ and $p_1 - p_0 = 1$, and hence the line integral contains the simple numerical value, 1. Therefore, we find the simple result:

The increase per unit of time in the circulation of a curve which is the section of a solenoid by any given plane has the numerical value of unity. We have already determined the direction of this increase of circulation, and in order to distinguish the two opposite directions from each other we may designate the increase of circulation by $+1$ when its direction agrees with the direction chosen as positive for the movement along the curve and by -1 in the opposite case.

We easily pass from the result just found for the circulation of a curve, that is the intersection of a plane with a solenoid to the corresponding general theorem for any curve whatever. Through the given arbitrary curve we draw a surface which intersects all the solenoids inclosed within the curve. On this surface the solenoids determine a system of parallelogrammatic curves, each of which receives in the unit of time an increase of circulation of either $+1$ or -1. But according to the summation theorem No. 2 for line integrals, the line integral along the exterior contour is equal to the sum of the line integrals along all individual contours, and, therefore, is simply equal to the number of the included solenoids if all turn in the same direction, otherwise it is equal to the excess, A, of the number of solenoids turning positively over the number turning negatively. Since this line integral is equal to the increase per unit of time in the circulation, C, of the curve under consideration, we can, therefore, express the result by the formula

$$A = \frac{dC}{dt} \tag{6}$$

If we express the enumeration in question algebraically we can consider the number, A, with its algebraic sign, simply as the number of solenoids inclosed within the curve and can express the result by the following theorem:

The increase in a unit of time in the circulation of any given *closed curve is equal to the number of solenoids inclosed within the curve.*

With the help of this theorem we can follow the variation with time of the varying value of the circulation of a closed chain of fluid particles, provided that we know at every moment the courses of the isobaric and isosteric surfaces. The number, A, will vary continually for two reasons: First, because the curve is in motion, and, second, because the isobaric and isosteric surfaces vary in consequence of the varying form and location of the conditions as to density and pressure, so that the curve incloses a bundle of solenoids that is continually varying.

IV.—THE MOST IMPORTANT CIRCULATORY MOVEMENTS OF THE ATMOSPHERE.

We have already called attention to the general course of the isobaric and isosteric surfaces in the atmosphere. In general, these surfaces succeed each other quite accurately because the density in general increases and diminishes with the pressure. They would be absolutely parallel if the density were a function of the pressure only. In that case the two systems of surfaces would not intersect each other and no solenoids would be formed. Under these circumstances the circulation of a curve in the atmosphere could be neither accelerated nor retarded, but would be a constant characteristic of the curve. This is the well-known result to which we arrive as the basis of the Helmholtz-Kelvin theory.

However, the density or the specific volume of the air is never a function of the pressure only, but also depends on the variability from point to point of the temperature and moisture. Since the influence of the moisture on the specific volume of the air is unimportant we will in the following qualitative study, for the sake of simplicity, consider only the temperature. We have then to recall that when the temperature is high the specific volume of the air is greater than would be expected for the given pressure, and when the temperature is low the specific volume is smaller. Hence in hot regions we shall have at the surface of the earth the same specific volumes of the air that in colder regions are to be found only in the higher layers of air. Therefore, the isosteric surfaces must deviate from the isobaric surfaces, and always in such a way that in hot regions they are lower, in cold regions higher than the corresponding isobaric surfaces. Therefore, the two sets of surfaces must necessarily intersect each other and form solenoids that cause a circulatory motion of the atmosphere. The general nature of this circulatory motion is easily deduced from the known distribution of pressure and temperature with the help of our fundamental theorem.

First, we may disregard all seasonal and diurnal variations of temperature and pressure, and all irregularities of a local nature arising from the distribution of land and ocean, or from the nature of the earth's surface. Therefore the pressure will be quite uniformly distributed over the whole globe, and will show no important differences in the polar and the equatorial regions. Hence the isobaric surfaces will be almost exactly parallel to the earth's surface. On the other hand, the polar regions have a low and the equatorial regions a high temperature, so that the isosteric surfaces are elevated in the polar regions and sink toward the equator. The two sets of surfaces intersect each other and form solenoids that surround almost the whole earth like parallel circles. A meridional section through this system of solenoids is illustrated by fig. 4, in which, as in all the subsequent figures, the isobars are represented by fine and the isosteres by heavy lines and the altitude of the atmosphere is much exaggerated. The gradient, G, is directed vertically upwards, the vector of motion, B, on the other hand, is inclined somewhat toward the equatorial side, and the acceleration of the circulation directed from the vector of motion toward the gradient will produce a circulation by virtue of which the air at the earth's surface flows from the poles toward the equator, where it ascends and then again flows toward the poles only to sink again in higher latitudes. This is the well-known general circulation between the poles and the equator which, especially in the trade-wind zone, appear as a regular well-developed movement.

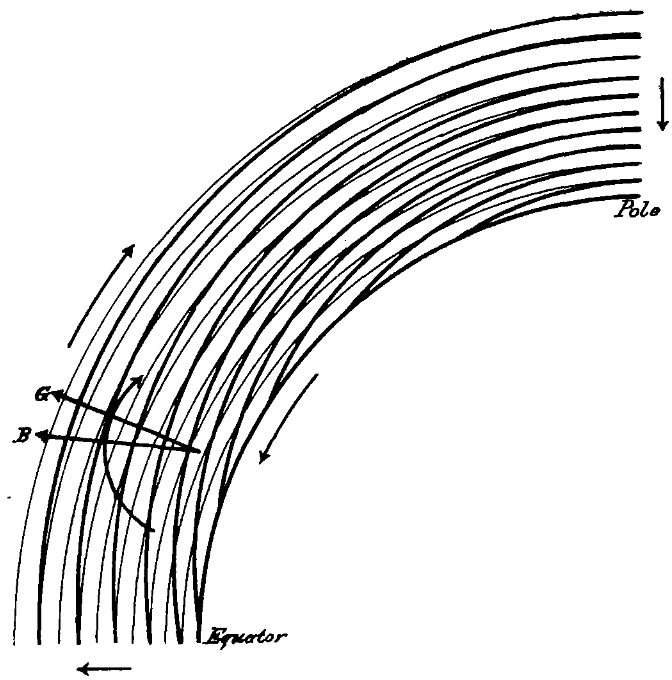

FIG. 4.

Connected with this broad, steady circulation, there is a series of smaller movements of periodic nature, which all arise from the seasonal and diurnal variations of temperature in connection with the irregular peculiarities of the earth's surface. The importance of the peculiarities of the surface of the earth depends upon the fact that the atmosphere is only to a slight extent warmed by direct insolation and only slightly cooled by direct radiation. It is at the surface of the earth that the large variations of temperature occur in consequence of insolation and radiation, and thereby the adjacent strata of air are warmed or cooled indirectly. Therefore, the ranges of temperature in this stratum of air vary according to the peculiarities of the surface of the earth.

In this respect the most important consideration is the difference between land and ocean. The land is warmed by insolation and cooled by radiation more quickly than is the ocean. Therefore, the air over the land is warmed more by day and cooled more by night than the air over the ocean. The isosteric surfaces during the daytime are, therefore, relatively high above the ocean and relatively low above the land; they must, therefore, intersect the horizontal isobaric surfaces and form a system of solenoids that follow along the coasts.

A section through this system of solenoids is illustrated by fig. 5; the acceleration of the circulation directed from the vector of motion toward the gradient induces a circulation by reason of which the air at the surface of the earth flows from the sea toward the land, where it rises, and, after flowing backwards, sinks again to the sea. At night time everything is reversed; the isosteric surfaces then lie higher over the land than over the sea; the solenoids change their signs and induce circulation in the opposite direction. Thus we observe the well-known phenomena of the land and sea winds.

The seasonal change of temperature makes itself felt in the same way as the diurnal change. In summer the isosteric surfaces over the continents are in general lower and over the oceans higher than the corresponding isobaric surfaces. The solenoids thus formed along the coast produce a circulation in which the wind at the surface of the earth has on the average a direction from the sea to the land rather than the contrary. In winter the isosteric surfaces over the continents are on the average higher and over the oceans lower than the corresponding isobaric surfaces; the solenoids lying along the coast have opposite signs and induce a circulation in which the wind at the surface of the earth is directed principally from the land toward the sea. Thus we arrive at the well-known phenomena of the monsoon winds.

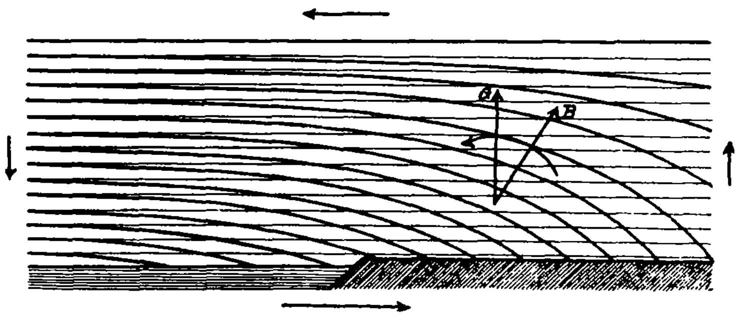

FIG. 5.

In addition to the distribution of land and water, the orography of the earth's surface comes into consideration. The strata of air warmed by insolation or cooled by radiation have the same form as that of the surface of the earth before the conditions are modified by the motions of the air. Above a horizontal plane surface the air strata have the form of a horizontal disc, and the isosteric surfaces, notwithstanding their rise and fall in consequence of the change of temperature, retain the form of horizontal discs so that they can never intersect the isobaric surfaces which also lie as horizontal planes. On the other hand, on the declivity of a mountain the strata of air, warmed by insolation and cooled by radiation, have an inclined position. In the daytime when this layer is warmed more than the surrounding air, the isosteric surfaces, which are horizontal planes at a great distance, sink lower if they, when prolonged, intersect this layer and cut the isobaric surfaces that lie as horizontal planes.

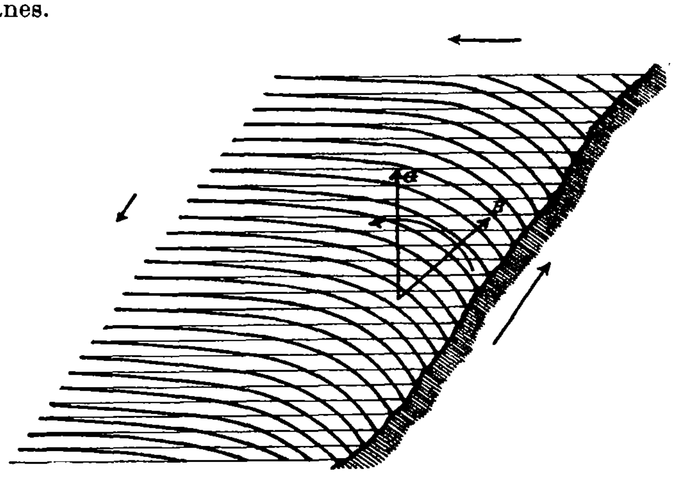

FIG. 6.

Along the slope of the mountain a system of solenoids will be formed, a section of which is illustrated by fig. 6. The acceleration of circulation directed from the vector of motion to the gradient will induce a circulation in which the air ascending along the slope, rises above the summit of the mountain only to flow back at some upper level and sink down again at a greater distance. At night time this layer of air is colder than the rest of the air; the isosteric surfaces, which at greater distances lie horizontal and plane, lie higher within this layer; the solenoids have the opposite sign and produce an opposite acceleration; therefore the cold air flows downward and sinks to the bottom of the valley, while the air pushed thence, ascends and gradually replaces the air that has flowed away at higher altitudes. This explains the day and night winds that occur regularly in mountainous countries, where the day wind is directed from valley to mountain top, and the night wind from the mountain down to the valley.

This latter phenomenon is more pronounced in proportion as the mountain is larger. On the other hand, the smaller the irregularities on the surface of the earth by so much the feebler the intensity and more irregular the course of the solenoids will be. Without causing important winds in definite directions at the surface of the earth, these solenoids will induce local ascending currents of air irregularly distributed, which in fine weather are the causes of the formation of cumulus clouds. So long as the ascending masses of air are warmer than the surrounding air the isosteric surfaces will have depressions where they pass through these masses

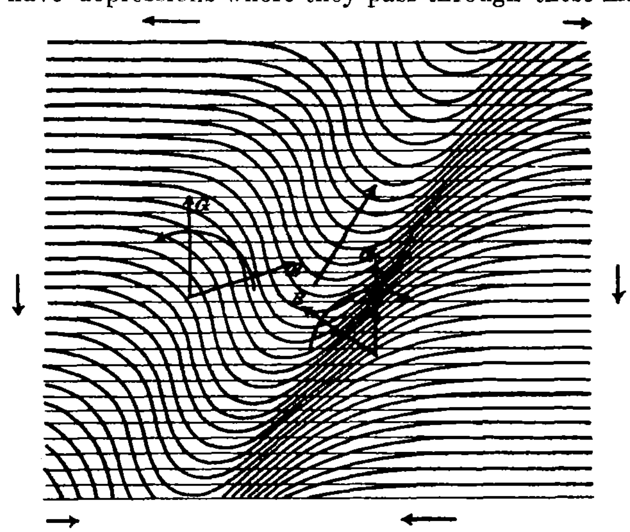

FIG. 7.

of air. A vertical section through such a column of ascending warm air is illustrated in fig. 7, while in fig. 8 is illustrated the extreme case in which a separate mass of air is so strongly

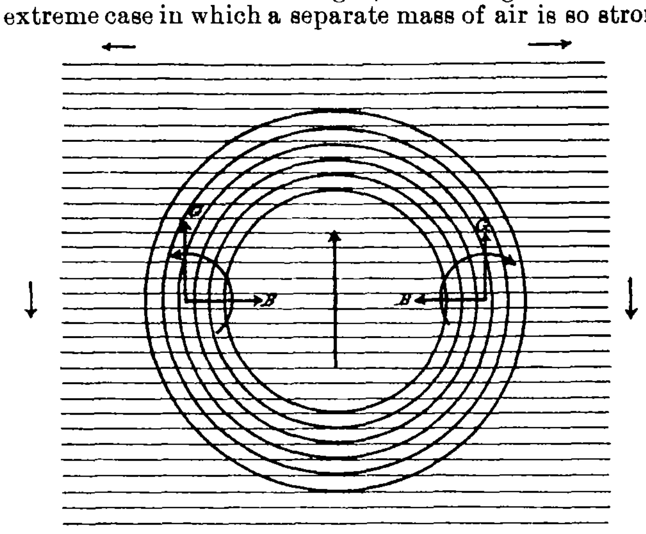

FIG. 8.

heated that its specific volume is even greater than that of the air lying vertically above it. This mass of air is therefore surrounded by closed isosteric surfaces which are here drawn as circles. In order to simplify the drawing, the variation with altitude of the specific volume of the surrounding air is disregarded. In both cases the circulatory acceleration, directed from the vector of motion toward the gradient, will produce a circulation in which the interior light masses of air must rise relatively to the exterior heavy air. In the last case, in which the isosteric surfaces are closed, this ascending movement also results as a consequence of the law of Archimedes relative to buoyancy. This latter law can, therefore, be considered as a peculiarly special case of our law of circulation.

The influence of the rotation of the earth is only slightly felt in the land and sea winds or mountain and valley winds which depend on the alternation of day and night, because we have here rapidly changing directions of motion, so that the deflecting force of the earth's rotation can have no long-continued accumulative effect. But we can imagine conditions to exist by virtue of which the air over a large area of the earth may, during many days be heated more than the surrounding region. As a consequence of the insolation this will occur most easily over extended plains where the ventilation due to the local ascending currents just considered is but slightly effective. On the ocean the warm ocean currents surrounded by cold water can cause such a warming of the superincumbent air as will continue day and night without interruption. Within this warm mass of air the isosteric surfaces become depressions. The isobaric surfaces can also simultaneously contain depressions, in consequence of diminished weight and the consequent smaller pressure of the warm masses of air. But the depressions of the isosteric surfaces will be the larger because these surfaces, in consequence of diminished pressure must sink precisely as much as the isobars, and because to this depression that which depends on the higher temperature must still be added. The isosteric surfaces must, therefore, intersect the isobars and form a system of solenoids which will surround the hot masses of air like a ring. A section through this system of solenoids is illustrated by fig. 9, and shows that the circulatory acceleration produces a circulation directed from the vector of motion toward the gradient, in which the masses of air flowing from all sides along the earth's surface rise in the central regions, and higher up flow away only to descend again at a great distance.

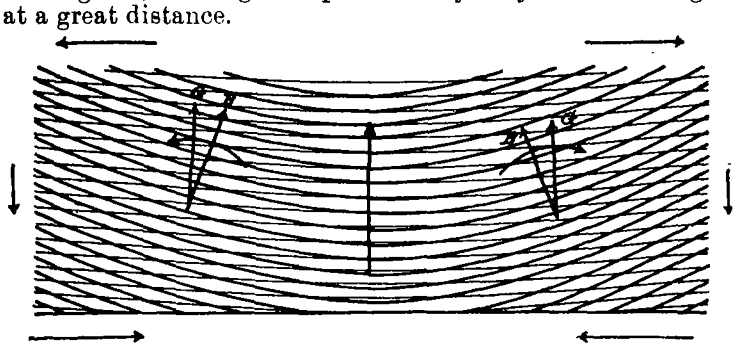

FIG. 9.

The general movement of the atmosphere is quite analogous to that just considered in figs. 7 and 8, except that the preliminary assumptions differ in two particulars; the heated mass of air has a much greater extension, and the conditions are not such that all are necessarily reversed during the night whether it be that the heating takes place over the land, by insolation, or over the sea, by warm ocean currents. In the resulting long-continued movement of the air over great distances the deflecting force of the earth's rotation makes itself felt and the original radial inflow of the air below and the corresponding outflow of the air above are turned into movements of a spiral nature. Therefore, the rotation in horizontal planes is superimposed on the original circulation in vertical planes and the friction *alone* sets a limit to the intensity of the two movements. When the movement has attained such an intensity that the motion is large in comparison with the forces that cause it then the conditions are such that for a first approximation we are justified in making application of the Helmholtz-Kelvin vortex theorems; the vortex that is formed will then, so to speak, endeavor to retain its individuality and can only slowly change by the forces that act upon it to increase or destroy the vortex. Therefore, when the conditions are favorable thereto the whole mass of air under consideration can be carried onward by the general atmospheric currents while retaining its own vortex motion. A progressive movement of the vortex can also be brought about, in that the center itself of the whirl-forming forces progresses, and, therefore, alongside of the old whirl there goes on a more or less continuous development of a new one. Such a transfer of the center of the whirl-producing forces becomes possible as soon as the movement [of the whirl] has progressed so far that the warm air present within the center is no longer the air that has been warmed *in situ* by reason of the given local conditions, but is air that has flowed in from without, for the air flowing in from different sides will, in general, have correspondingly different temperatures, and, on account of this want of symmetry, the place where the air is hottest, and where, therefore, the isosteric surfaces have their greatest depressions, will not coincide with the momentary center of the whirl. The system of solenoids will, therefore, move and a new whirl will form alongside of the old one and unite with it to form a whirl somewhat further forward. With the new whirl the same process is repeated, and we thus get a vortex advancing from one place to another, as we observe in the case of cyclones.

A local surplus would arise because of the many ascending currents in the atmosphere if there were not also corresponding descending movements. This descending movement can either be distributed uniformly over large areas, and be, therefore, less noticeable, or it can be localized in more definitely limited descending currents of air. This latter will occur with special ease when great masses of air that are colder, and, therefore, denser than the surrounding air, have collected in the upper strata of the atmosphere. On account of the greater pressure of the denser masses of air the isobaric surfaces will be higher in this region; the isosteric surfaces will also at the same time be higher both because of the increased pressure, causing a rise that is the same as that of the isobaric surfaces, and a further addition because of the contraction in consequence of the lower temperatures. The isobaric and isosteric surfaces must, therefore, intersect each other and form solenoids that surround the cold masses of air. Fig. 10

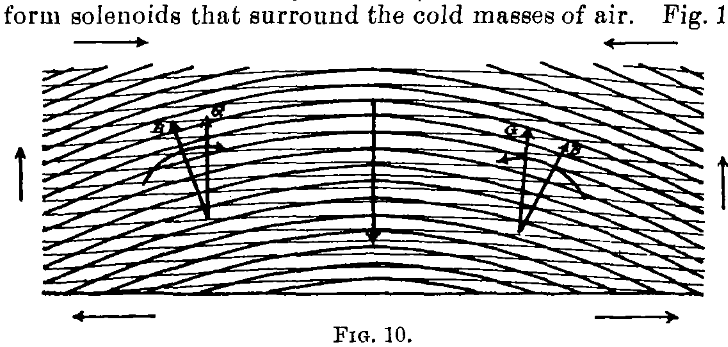

FIG. 10.

represents a section through such a system of solenoids after the movement has continued for some time so that the denser masses of air have descended to the surface of the earth. The acceleration of circulation directed from the vector of motion, B, to the gradient, G, will give rise to a circulation in which the air in the higher strata flows inward from all sides, sinks down in the central region, flows slowly along the earth's surface, and ascends again at some distance. If the circulation continues for a long time, then the radial inflow above as well as the radial outflow below will change into movements of a spiral nature in consequence of the influence of the rotation of the earth. In this way we arrive at the phenomenon of the anticyclone.

In the preceding development we have adopted the so-called physical theory of cyclones and anticyclones, according to which an ascending current of warm air is considered as the primary cause of the formation of a cyclone and a descending current of cold air as the primary cause of the formation of an anticyclone. It is well known that another theory has also been advanced, the so-called mechanical theory, according to which the primary cause is to be sought in the collision between the great atmospheric currents in the upper strata of air; or the cyclone and anticyclone are to be considered as formations on the edges or boundaries of the great circulations of the atmosphere. So far as I know, no accurate development of the theory of cyclones and anticyclones, based on this theory, has been published, and, therefore, we can not go into a positive discussion of it. On the other hand, we can formulate a criterion by which, through purely empirical investigations, we may decide to what extent the physical theory gives a satisfactory explanation of the mechanics of the formation of cyclones or anticyclones.

To this end we assume that we know the distribution of density, pressure, and wind at every moment since the formation of a cyclone or anticyclone. Therefore we can construct the isosteric and the isobaric surfaces and find the number of the solenoids present at every moment, and equally, from the observations of the wind compute the circulation along different closed curves at various times. If now the physical theory is correct, then the number of solenoids that are, or have been present must suffice to explain the existing circulation or velocity of the wind. Of course we can only make accurate computations when we also take into consideration the friction and the rotation of the earth. But if, for instance, we find that in a cyclone that has existed for three days, only so many solenoids have been present as could, ignoring friction, produce only a small percentage of the existing circulation in the course of three days, then we must conclude that other forces have been active in forming the cyclone as well as those represented by these solenoids. On the other hand, if we find that, omitting the rotation of the earth and the friction, the number of solenoids present is sufficient to produce the existing circulation or wind velocity in the course of a few hours, then we must conclude that during the three days a great excess of force must have been present in order to overcome during this time the resisting forces not explicitly considered. Whether this excess also precisely suffices can be decided only after completely taking account of the earth's rotation and the friction. Moreover, it is only after we have demonstrated the insufficiency of the motive forces that depend on the solenoids locally present in the cyclone, that we can have any reason to seek for other causes of formation of cyclones and take into consideration the more distant solenoids of the general atmospheric circulation.

This example has a special interest because it has to do, not with an impracticable ideal experiment, but with investigations that can be and, indeed, have already in part been carried out, although, in truth, no cyclone has as yet been completely investigated by means of simultaneous observations in the upper strata of the air. However, at Blue Hill they have succeeded in obtaining a section through a passing anticyclone and cyclone by observations, with kites, on four successive days, September 21–24, 1898. My pupil, Mr. Sandström, has constructed the isobaric and isosteric surfaces of this cyclone from the observations published by Mr. Helm Clayton, so far as was possible by combinations of the observations made on these different days and will, it is hoped, soon publish his work on this subject. I will here only remark that the number of the existing solenoids found in this manner is so great that they suffice to develop the strongest observed wind velocity in the course of a few hours. The great interest that is now shown in obtaining observations in the higher strata of the atmosphere leads us to hope that it will not be long before it becomes possible to follow the complete history of the development of a cyclone by means of systematic simultaneous observations at different places, instead of being compelled, as in the present case, to construct a hypothetical condition for any moment from observations made at different times. By means of such simultaneous observations we shall be able to decide between the mechanical and the physical theories of the progressive movement of a cyclone. If the physical theory is correct, so that a continuous new formation of whirls takes place near the old one, then the system of solenoids must somewhat precede the whirl proper; if, on the other hand, the cyclone is carried forward by the general atmospheric current, then the solenoid system, if one is present, will follow the whirl exactly.

In the preceding we have for simplicity considered the trades, monsoons, land and sea breezes, mountain and valley winds, cyclones and anticyclones as phenomena isolated from each other. But in fact a complete isolation of these systems of wind from each other is not practicable, but the actual (natural) winds always have more or less complex causes, and it is only in order to simplify this review of the subject, that we have made use of this schematic analysis into individual phenomena. Any such analysis will seem artificial in the direct application of this present theory to practical meteorology, where we observe the existing distribution of density and pressure in connection with the existing winds. No matter how complicated the conditions are we then have always to do with the real winds and their real causes. Thus this present theory differs fundamentally from the ordinary dynamic theories that are founded on the solution of special integrals of the equations of motion, and where one must first assume a general, farfetched idealization of actual conditions before the theory can be brought to apply.

If, therefore, one would study the motions of the atmosphere by using the theory here developed, then the problem would be to find the actual course of the isobaric and isosteric surfaces in the atmosphere, and the courses of the solenoids formed by these groups of surfaces. In this investigation we will only exceptionally, or in general never, find the ideal conditions above assumed. We shall never find solenoids that precisely follow the parallels of latitude, and, therefore, produce pure trade winds. Quite as rarely shall we find solenoids that follow the coasts, precisely for long distances, and, therefore, produce a pure land and sea wind. We shall rather find that the actual solenoids generally encircle the whole globe as tubes or curves of rather irregular form, and that they generally have more or less decided changes of direction in the passage from land to sea, and, moreover, are always in motion with the change from day to night, and from summer to winter. During the daytime, or the summer, the solenoids over the land deviate toward the polar side; during the night, or in winter, they deviate toward the equatorial side. If we make these actual solenoids the basis of our study we gain the advantage that we see the actual winds in connection with their actual and complete causes. For example the Indian monsoon will thus be seen to be neither a pure land and sea breeze nor a pure trade wind, but a combination of land and sea wind and trade wind, as it really is.

For similar reasons one must not expect to see the circular system of solenoids above mentioned always perfectly developed in the cyclone and anticyclone. For, simultaneous with the local elevation of temperature in the central region of the cyclone, we have to consider a general diminution of temperature from the equator toward the pole. Therefore, the isosteric surfaces in which local depressions appear do not lie parallel to the earth's surface, but are depressed toward the equator. Therefore, the intersections of these with the isobaric surfaces that run approximately parallel to the earth's surface, as also the corresponding solenoids will, when projected on the surface of the earth, appear as shown in fig. 11. Most solenoids belong to the solenoid system that en-

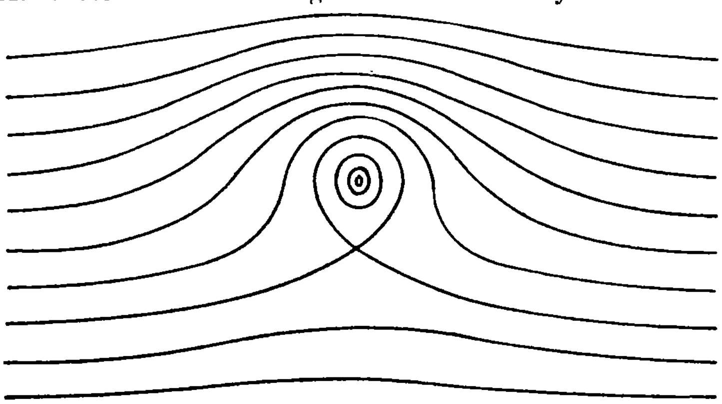

FIG. 11.

circles the whole earth; only in the cyclonic region do they have a deviation toward the polar side. Circular solenoids inclosed within the cyclones occur only in the central region, and only when the depressions in the isosteric surfaces are sufficiently deep. All other solenoids run as in fig. 12 where

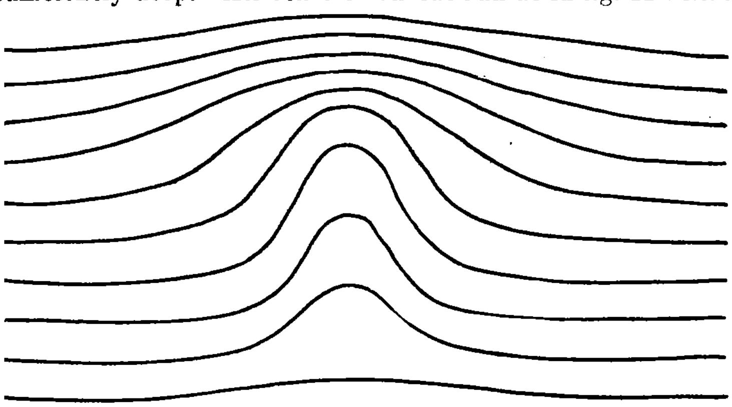

FIG. 12.

all encircle the whole earth, but have the above-mentioned bend within the cyclone region. In the study of these actual courses of solenoids in the cyclones we, therefore, see the winds of the cyclone in connection with the general circulation of the atmosphere. Here it should be remarked that the ordinary progress from west to east of the cyclone in our latitudes can in general be so represented that these bends advance like waves in the solenoid system encircling the whole globe.

Quite similar remarks apply also to the anticyclones. If the physical theory of the anticyclone is correct so that they contain colder air than that of their immediate surroundings, then the solenoids surrounding the whole earth must show bends (curves) in the anticyclonic region which in opposition to the bends in the cyclonic regions must be directed toward the equator.

V.—CONCLUDING REMARKS.

In the preceding we have utilized our fundamental theorem for the purely qualitative discussion of the most important atmospheric movements. But the theorem itself is a quantitative one and, therefore, allows of an accurate quantitative investigation of the phenomena. However, it would certainly be premature to immediately use the theorem as the basis of extensive computations of atmospheric movements. The formal imperfection due to the fact that we have not considered the rotation of the earth and the friction would, for the present, prevent the numerical application. But this omission is not difficult to remedy so far as concerns the mathematics. The two generalizations already indicated in the introduction as necessary, where we take into consideration the friction and the rotation of the earth consist simply in this, that we supplement the right-hand member of the fundamental equation (6) by two terms, the first of which is the line integral of the frictional forces taken along the curve, and the second is the line integral of the deviating force due to the earth's rotation. But the most important difficulties are first met with in the applications themselves. For the frictional resistance depends on the relative velocities of particles of air lying near each other and a computation of the frictional resistance based on a rational principle would, therefore, demand a knowledge of the movement of the air, not from degree to degree, but from millimeter to millimeter. This circumstance shows that we must necessarily seek another way and that the indicated theoretical generalization will not have so great a practical importance as at the first glance we should have expected.

The value of the theory here described therefore does not consist especially in the formal possibility which it opens up of numerically following the atmospheric movements. Its great importance is rather to be sought in the fact that the theory gives a rational dynamic principle by which the facts of observation can be grouped. In this way we shall also provide the best foundation for a future quantitative dynamic meteorology. The problem will, therefore, always be simply this, to record the number and location of the solenoids and the corresponding distributions and intensities of the wind. We shall then, through experience instead of computation, learn how to take into consideration the earth's rotation and the atmospheric friction. In the case of periodic winds of short periods, such as the land and sea breezes or the mountain and valley winds that follow the alternations of day and night, we shall probably find that the actual circulation does not vary much from the values that are computed by our theorem, which neglects the rotation of the earth and the friction, since in these cases the work done by the solenoids consists essentially in overcoming the inertia of the masses of air. On the other hand, the study of the number of solenoids and the strength of the winds in the case of periodic winds of long period like the Monsoon, or the steady winds like the Trades, will probably lead us to a knowledge of the conditions of equilibrium between the moving forces represented by the solenoids and the resistances that arise in consequence of the state of steady motion. In cyclones we shall have occasion to study three stages: that of accelerating motion, where the inertia is the important resistance; that of steady motion, where the solenoids just suffice to maintain the movement that has been produced against the resisting forces; finally, the diminishing movement, where the resisting forces are overpowering. An accurate knowledge of cyclones from this point of view may be of special importance for weather prediction. From the number of solenoids we may conclude to what extent the wind will increase or decrease in intensity in the immediate future. Everything depends simply on whether we can obtain a sufficient number of systematic observations from the upper strata of air, and the technical details of this class of observations are already so far developed that it can no longer be doubted that the observations may be obtained with such regularity that they can be utilized in daily weather predictions.

In this connection I will further remark that the theory here developed can be applied to the movements of the ocean just as to those of the atmosphere. In the ocean the temperature and the saltness play the same part in changing the density as do the temperature and the moisture in the case of the atmosphere. Eventually the theory also retains its applicability when we consider the atmosphere and the ocean together as one fluid medium. This is of great importance because of the extensive interaction between the movements of the air and of the ocean. Hence an excellent opportunity for the simultaneous solution of great meteorological and hydrographic problems will be afforded if the plans projected at the Hydrographic Congress in Stockholm in 1899 can be realized, so that the hydrographic expeditions sent out many times yearly by the participating nations can also carry meteorologists with instruments for the investigation of the upper strata of the air. In this respect the North Atlantic Ocean in the autumn and winter will offer especial interest. Perhaps it will here be possible to study the development of cyclones that probably often form over the region of the Gulf Stream, and therewith simultaneously measure the quantity of heat given out by the ocean and consumed in cyclonic formation.

THE PORTO RICAN HURRICANE OF 1899.

By C. O. PAULLIN, Nautical Expert, United States Hydrographic Office.

Soon after the occurrence of the Porto Rican hurricane of 1899, the United States Weather Bureau published a complete account of the passage of this storm through the West Indies and along the American coast. The daily maps of conditions over the Atlantic Ocean, compiled by the United States Hydrographic Office from the reports of its voluntary observers, make it possible to furnish some additional information of exceptional interest to meteorologists concerning this storm, both previous and subsequent to the period of its history covered by the Weather Bureau.

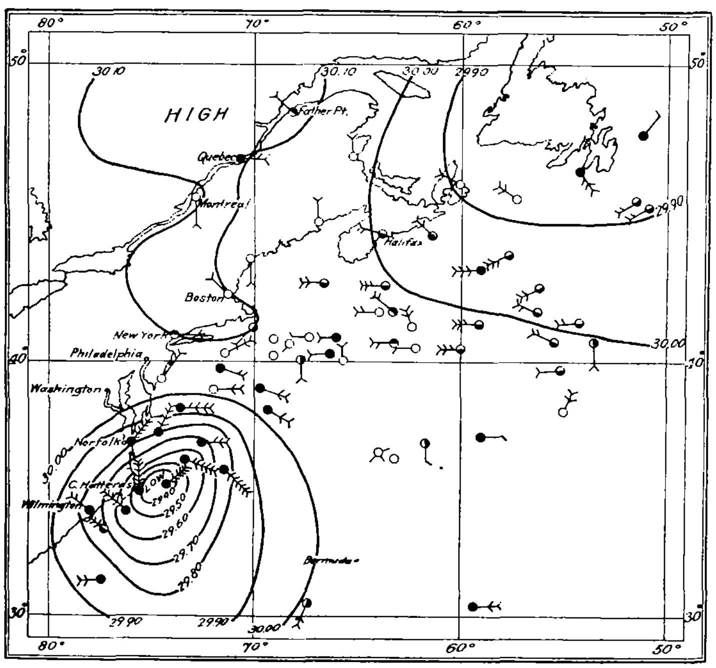

FIG. 13.—Greenwich noon, August 17, 1899.

The tropical storms of the North Atlantic generally originate to the eastward of the Lesser Antilles within the belt of calms which covers the ocean from latitude 5° to 15° north. Owing to the scarcity of observing vessels in this part of the Atlantic, and the relatively small area which the hurricane here attains, reports of these storms to the eastward of the fiftieth meridian are seldom received. Information concerning tropical storms at or near their place of origin is, consequently, almost wholly lacking, and much interest attaches to the report of the British steamship *Grangense*, which vessel encountered the late hurricane 1,800 miles east by south of the Island of Guadeloupe. The *Grangense* passed through the center of the storm and took very careful and complete observations, warranting the publication of her log in full, as follows:

At noon of August 3, when in latitude 11° 51′ north, longitude 35° 42′ west, we experienced a sudden change in the weather, which, being most unusual in this part of the world, is worthy of note. Early in the afternoon the barometer began slowly to fall from 29.93 inches. At 2 p. m. it stood 29.73, the sky becoming overcast with cumulo-nimbus clouds and the wind freshening to a moderate gale from north-northwest. At 4 p. m. the barometer read 29.53 inches, the wind remaining from the same direction with force increased to a fresh gale, accompanied with heavy rain. At 5 p. m. the barometer reached its lowest reading, 29.38 inches, while the wind fell calm and the rain ceased; very heavy nimbus clouds traveled overhead at a high speed from the southwest and a high, short, and dangerous sea from the northeast, caused the ship to pitch heavily and made it necessary to let her head fall off to the east in order to make headway, the ship being very light. At 6:30 p. m. a light breeze came out of the south-southwest and the barometer rose to 29.43 inches, clearly indicating that the center had passed. At 7 p. m. the wind increased to a strong south-southwest gale, with excessive rain beating down the northeast sea and enabling us to return to our course, northeast one-quarter east. At 8 p. m. the barometer stood at 29.58 inches, with a moderate gale hauling gradually southward. After two heavy squalls at 10 p. m. the weather cleared; barometer 29.73 inches, steadily rising; sea coming up from south-southeast; sky clearing and stars shining out again; strong breeze hauling to east. And so finished this little storm which showed all the symptoms of a genuine West Indian hurricane undeveloped, with the exception of the sea in the vortex, which, instead of being confused, came almost suddenly from the northeast, and remained from that quarter until the wind and sea from the receding semicircle overwhelmed it. Captain Spedding, who has been in this particular trade, from Europe to the river Amazon, for many years, and many others on board who have been long acquainted with these regions, say they have never experienced any weather of a cyclonic character so far to the eastward before.

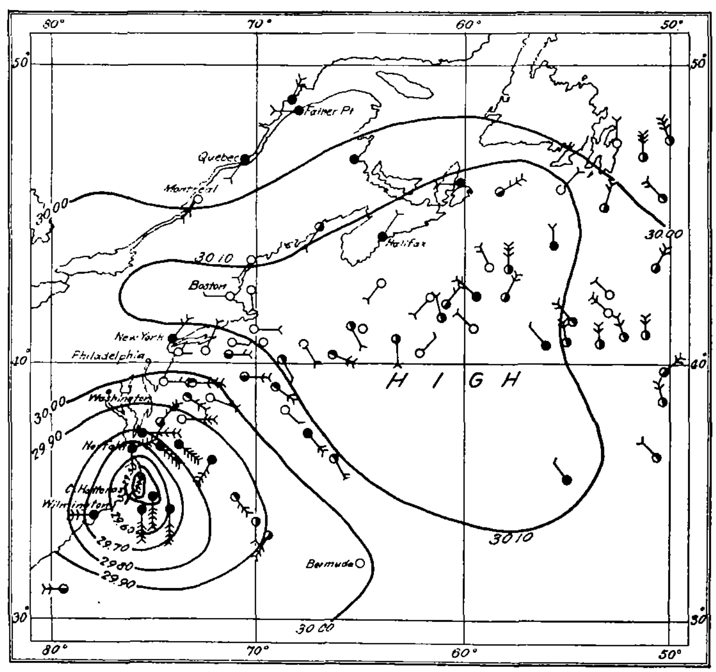

FIG. 14.—Greenwich noon, August 18, 1899.

From the foregoing log it appears that when the *Grangense* encountered the hurricane its development was not complete. The exceedingly low barometer which characterizes the tropical storm in its maturity was lacking, and neither the winds nor the sea had as yet attained dangerous violence. At the same time, according to the above account, this storm